Sugar Maple Height–Diameter and Age–Diameter Relationships in an Uneven-Aged Northern Hardwood Stand

Laura S. Kenefic, *USDA Forest Service, Northeastern Research Station, R.R. 1, Box 589, Bradley, ME 04411 and* **Ralph D. Nyland,** *State University of New York College of Environmental Science and Forestry, 1 Forestry Drive, Syracuse, NY 13210.*

ABSTRACT: *Sugar maple (*Acer saccharum *Marsh.) height–diameter and age–diameter relationships are explored in a balanced uneven-aged northern hardwood stand in central New York. Results show that although both height and age vary considerably with diameter, these relationships can be described by statistically valid equations. The age–diameter relationship compares favorably to one reported by Tubbs (1977) for sugar maple in unmanaged (virgin) northern hardwoods, suggesting that periodic cuttings improved growing conditions in our stand. Deliberate attempts to control size-class distribution and tree spacing should continue to increase diameter growth rates and decrease the time needed to reach certain threshold tree sizes. Growth rates that can be reasonably expected in this and similarly structured stands are provided. Lastly, a wide range of heights and diameters are documented, confirming the structural complexity associated with the balanced selection system. An equation to predict sugar maple height from diameter is provided and may prove useful when assessing habitat or visual characteristics of complex uneven-aged stands. North. J. Appl. For. 16(1):43–47.*

Foresters have long recognized the correlation between tree diameter and age in uneven-aged stands (Leak 1985, Betters and Woods 1981, Tubbs 1977, Gates and Nichols 1930). Correlations between tree diameter and height, or age and height, have also been implied (Nyland 1996). Height–diameter equations have traditionally been used to estimate height when calculating tree volume and have proven valuable for describing trends in tree and stand growth. Appropriate equations demonstrate the degree of correlation between diameter and total height and give foresters a means to objectively portray canopy characteristics within structurally complex communities. A valid height–diameter equation could even be used to quantify vertical structure from stand table information, thus allowing practitioners to easily describe and assess habitat or visual characteristics of a stand.

The distribution of foliage from ground level to the tops of the tallest trees in managed uneven-aged northern hardwood stands dominated by sugar maple (*Acer saccharum* Marsh.) suggests a wide range in both tree diameter and height (Kenefic and Nyland 1996). In this paper, we test the hypotheses that relationships exist between sugar maple diameter and age, and diameter and height, in a managed uneven-aged northern hardwood stand. We examine a variety of equations to identify the best ones for expressing these relationships and discuss the implications of our findings with regard to managing and describing uneven-aged northern hardwoods.

Study Area

Data were collected from an uneven-aged northern hardwood stand on the Cuyler Hill State Forest, south of Syracuse in central New York. Sugar maple dominates this 12 ha stand, accounting for 65% of the basal area after the first selection treatment in 1973 and 59% after the second treatment in 1993. The 1994 residual stand also had American beech (7%), white ash (12%), black cherry (8%), and small amounts of yellow birch, basswood, red maple, eastern hemlock, eastern hophornbeam, and striped maple (collectively 14%). Trees sampled by increment borings at breast height showed a wide range of ages: sugar maple ($n = 96$) from 8 to 126 yr old; white ash ($n = 66$) from 21 to 123 yr; and black cherry ($n = 66$) from 23 to 88 yr.

Crown breakage during ice and snow storms has affected the heights of many trees, and logging has also damaged the crowns of some (Nyland 1986, Nyland et al. 1976). Both effects preclude meaningful estimation of site index and add variation to the relationship between tree height and diam-

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NOTE: Laura S. Kenefic is the corresponding author—Phone (207) 866-7258; Fax: (207) 866-4602; E-mail: lkenefic@apollo.umenfa.maine.edu. Reviews of an earlier version of this manuscript were provided by Douglas Maguire, Oregon State University; Robert Seymour and William Halteman, University of Maine; and Gerald Walton, USDA Forest Service. Funding for this research was provided by the McIntire-Stennis Cooperative Forestry Research Program and other sources of the SUNY College of Environmental Science and Forestry.

eter. Damaged trees were not excluded from the height–diameter study, as our objective was to develop a model for the actual (e.g., potentially ice- or logging-damaged) population of sugar maple trees in managed uneven-aged northern hardwood stands.

The stand sampled for this research is an old farm woodlot which apparently received periodic partial cuttings until the early to mid 20th century. In 1973, a single-tree selection cutting was applied to create a theoretically balanced structure conforming to the Arbogast (1957) reverse-J diameter distribution (Figure 1). A second treatment in 1993 re-created the reverse-J structure by reducing surpluses across the diameter distribution while protecting deficient size classes. Noncommercial and disease-prone species, unacceptable growing stock, and trees beyond maximum diameter were removed where possible without creating large openings in the stand. Each cutting left a good mixture of size and age classes throughout.

After the 1973 cutting, age and diameter data were collected for trees throughout the stand, including 96 sugar maples. Trees were sampled at random locations along transects and cored at 1.37 m above ground for determination of age at breast height (*A*). For the 1993 sampling of sugar maple height and diameter, we subjectively chose plots across a previously established 30.5 m grid to provide an adequate representation of stand and topographic conditions. At each of 42 sample points, diameter at breast height (*D*) and total height (*H*) were measured for the nearest three sugar maple trees greater than 1.8 m in height. Total height was measured on small trees less than 10 m tall using a graduated telescoping height pole. Taller trees were measured using a Blume-Leiss altimeter (hypsometer). A total of 127 trees were sampled in 1993 and 1994 (4 trees were sampled on 1 plot).

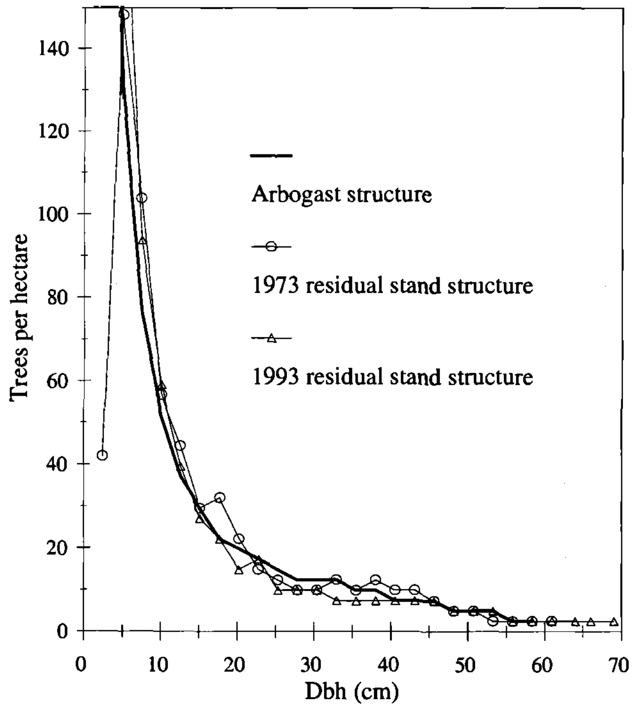

Figure 1. Diameter distribution of the study stand following selection treatment and the structural goal recommended by Arbogast (1957).

Regression Analysis

We compared equations of the form

$$y_i = f(\beta \mid x_i) + \epsilon_i$$

where

$$\epsilon_i^{iid} \sim N(0, x_i^n \sigma^2)$$

with y_i = predicted height or age, x_i = observed diameter, and n = weighting factor, to explore the relationship between individual tree diameter and age, and tree·diameter and height. Height–diameter equations include linear functions recommended by Curtis (1967) as well as nonlinear functions recommended by Huang et al. (1992) (Table 1). Both publications present a thorough summary of the height–diameter literature, and serve as excellent sources of potential equations. Additional nonlinear asymptotic equations were adapted from Seber and Wild (1989) and Parresol (1992).

Equations were weighted by D^n with $n = 0, -1, -2$, and -3 in order to identify the optimal weighting factor to correct for heteroskedasticity. Generalized R^2 (Kvålseth 1985) was calculated on the original scale using the corrected sum of squares $(1 - ssresid/\Sigma(y_i - y_m)^2)$, where the y_i's are the individual sample values and y_m is the sample mean. Furnival's (1961) index of fit (FI), a modified likelihood criterion that allows concurrent evaluation of root mean square error (RMSE), normality, and homoskedasticity, was used to identify optimal model form. FI has the advantage of simultaneously allowing comparison both across model forms and within models across weighting factors. The lower the FI value, the better the fit based on the criteria listed above.

Only equations with all significant parameters ($\alpha = 0.05$) were considered, and plots of standardized residuals against predicted variables were used to verify homogeneous variance. Rational biologic behavior was taken into consideration and evaluated using scatter plots of the sample data against the predictor equations.

Results

The age–diameter relationship was adequately described by the model $A = b_0 + b_1D + b_2D^2$ weighted by D^{-1}, with $b_0 = 4.5408$, $b_1 = 2.3618$, and $b_2 = -0.0125$ (Figure 2). One

Table 1. Linear and nonlinear height–diameter equations evaluated for sugar maple in an uneven-aged stand on the Cuyler Hill State Forest.

	Model	Reference
(1)	$H = b_0 + (b_1{*}D) + (b_2{*}D^2)$	Curtis 1967
(2)	$H = b_0 + (b_1{*}\text{Log}D)$	Curtis 1967
(3)	$\text{Log}H = b_0 + (b_1{*}D^{-1})$	Curtis 1967
(4)	$H = 1.3 + b_0{*}\exp[b_1/(D + b_2)]$	Huang et al. 1992
(5)	$H = 1.3 + b_0{*}(1 - \exp[-b_1{*}D])^{b2}$	Huang et al. 1992
(6)	$H = 1.3 + b_0{*}(1 - \exp[-b_1{*}D^{b2}])$	Huang et al. 1992
(7)	$H = 1.3 + b_0/(1 + (b_1^{-1}){*}(D^{-b2})$	Huang et al. 1992
(8)	$H = 1.3 + \exp[b_0 + (b_1{*}D^{b2})]$	Parresol 1992
(9)	$H = b_0{*}(1 - \exp[-b_1{*}D])$	Seber and Wild 1989

Note: Equations (4)–(8) are unit specific. Height (m) is determined from observed diameter (cm) adjusted by breast height 1.3m.

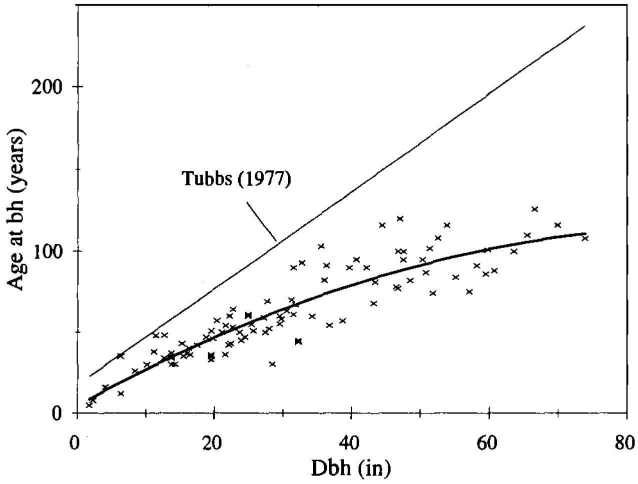

Figure 2. Age–diameter equation $A = 4.5408 + 2.3618*D - 0.0125*D^2$ weighted by D^{-1} plotted against the raw sample data, with Tubbs' (1977) equation.

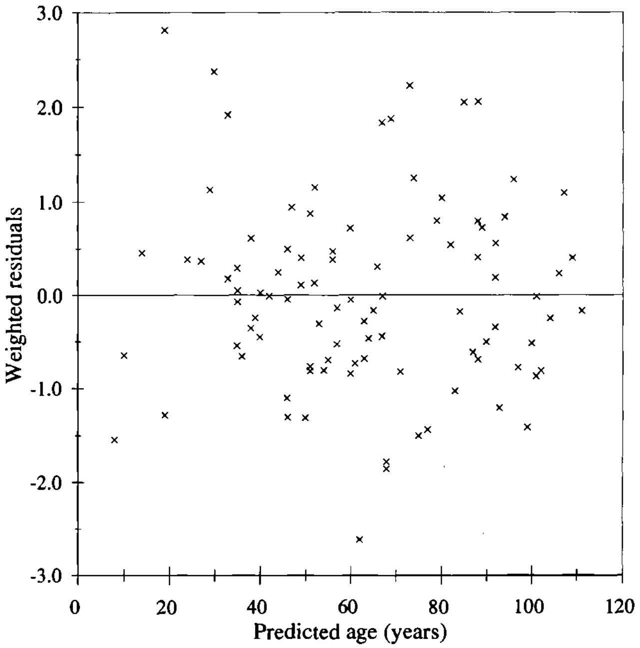

Figure 3. Plot of the weighted residuals versus predicted values for the suggested age–diameter equation.

outlier was removed based on residual analysis. This model satisfies the regression assumptions of fit, normality, and homogeneous variance (Figure 3), and is superior to other forms and weighting factors tested based on FI. The recommended function accounted for a high proportion of the variance in the age–diameter relationship ($R^2 = 0.81$). These findings suggest a general pattern of development with growth accelerating over time: trees reach 10 cm dbh at age 27 yr, 20 cm at 47 yr, 30 cm at 64 yr, 40 cm at 79 yr, 50 cm at 92 yr, and 60 cm at 101 yr.

Height–diameter equations weighted by D^{-1} consistently had the lowest FI values and best residual plots compared to log transformations or other weighting factors. Plots of the standardized residuals against predicted height for unweighted least squares regression showed either increasing variance or lack of fit. Linear functions with log transformation of D or H (Curtis 1967) resulted in high FI values relative to nonlinear equations weighted by D^{-1}. Table 2 provides a summary of regression output with FI values for all height–diameter equations weighted by D^{-1}.

The nonlinear functions (4–9) from Huang et al. (1992), Parresol (1992), and Seber and Wild (1989) gave similar results, with FI values varying more by weighting factor than by model form. Of the nonlinear models, Equation (4) had the lowest FI value (2.427), a finding that is consistent with that of Huang et al. (1992), who recommended the function for deciduous species in Alberta. R^2 was 0.92, and a graph of the equation against the sample data showed an excellent fit (Figure 4). The residual plot showed only slightly increasing variance over predicted height (Figure 5).

Other nonlinear equations proved viable, though not optimal. Equations (5) and (6), Chapman-Richards and Weibull-type functions also recommended by Huang et al. (1992), gave almost identical results. The first had $FI = 2.434$ and $R^2 = 0.92$, the second had $FI = 2.433$ and $R^2 = 0.99$. Residual plots for these two equations showed relatively constant variance. Graphs of the predicted lines against the sample data showed good fit, although both equations had a lower slope at large diameters than that observed for Equation (4). Equation (7), a modified logistic-type curve explored by Huang et al. (1992), also compared favorably with those listed above.

Equation (8) from Parresol (1992) was among the best five functions, with $FI = 2.568$ and $R^2 = 0.91$, but predicts lower heights in the middle diameter classes (10 to 40 cm) and slightly higher heights at large diameters (40+ cm). The residual plot suggests slightly increasing variance. Similarly, Equation (9), a nonlinear function from Seber and Wild (1989), performed reasonably well in terms of FI, but differs in shape from those above. Predicted height plateaus at approximately 40 cm dbh, with minimal increases at larger

Table 2. Results of regression analysis using height–diameter equations weighted by D^{-1}, including Furnival's (1961) index of fit.

	Model	R^2	RMSE	FI	b_0	b_1	b_2
(1)	$H = b_0 + (b_1*D) + (b_2*D^2)$	0.90	0.639	2.576	1.7644	1.1354	–0.0114
(4)	$H = 1.3 + b_0*\exp[b_1/(D + b_2)]$	0.92	0.602	2.427	36.8618	–15.4112	3.8011
(5)	$H = 1.3 + b_0*(1 - \exp[-b_1*D])^{b2}$	0.92	0.604	2.434	29.8127	0.0535	1.0429
(6)	$H = 1.3 + b_0*(1 - \exp[-b_1*D^{b2}])$	0.99	0.603	2.433	29.6130	0.0472	1.0357
(7)	$H = 1.3 + b_0/(1 + (b_1^{-1})*(D^{-b2})$	0.91	0.609	2.457	37.3561	0.0363	1.1197
(8)	$H = 1.3 + \exp[b_0 + (b_1*D^{b2})]$	0.91	0.637	2.568	4.5370	–4.3646	–0.3294
(9)	$H = b_0*(1 - \exp[-b_1*D])$	0.91	0.694	2.797	29.2691	0.0674	

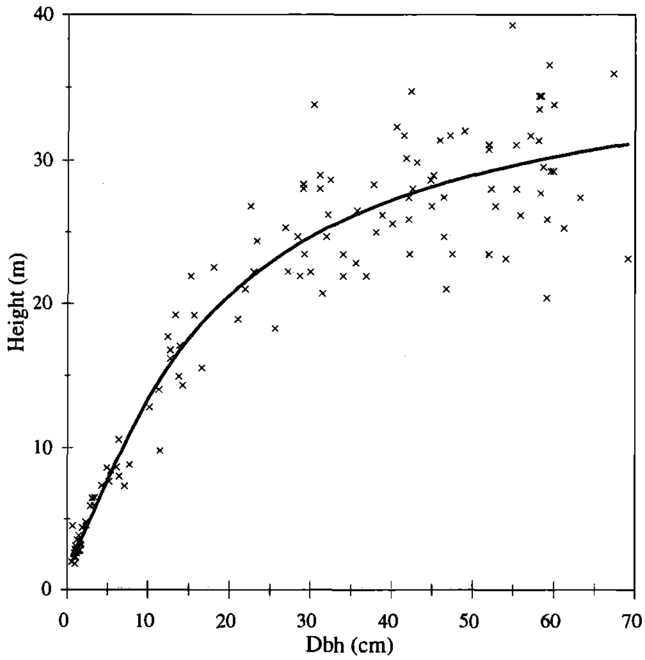

Figure 4. Height–diameter equation $H = 1.3 + 36.8618 * \exp[-15.4112/(D + 3.8011)]$ weighted by D^{-1} plotted against the raw sample data.

diameters. The residual plot suggests lack of fit. This function is not one historically used in height–diameter relationships and does not compare as well as those recommended by Huang et al. (1992).

Discussion

Sugar maple trees sampled in the study stand in 1973 showed a strong correlation between diameter and age. This relationship was adequately described by the quadratic model form used by Leak (1985) and supports the findings of his and other studies in uneven-aged northern hardwoods. Though not documented prior to 1973, past cuttings in our study stand apparently periodically triggered the regeneration of new cohorts and sustained the development of established trees. Furthermore, the 1973 age data suggest an acceleration of diameter growth as trees became larger, probably due to greater exposure of their crowns to direct overhead light. We have no evidence that the cuts made prior to 1973 represented single-tree selection system and suspect that the landowner took a more informal approach in which trees were removed largely from the sawtimber classes. More recent treatments that extended cutting down through the pole classes have reduced crowding among smaller trees and should enhance growing conditions for them. In fact, assessments in this stand 6 yr after the 1973 cutting (Mader and Nyland 1984) showed dramatic effects on the growth of trees less than 30.5 cm dbh, a result similar to that found by Eyre and Zillgitt (1953) in their pioneering work with the selection system in northern hardwoods.

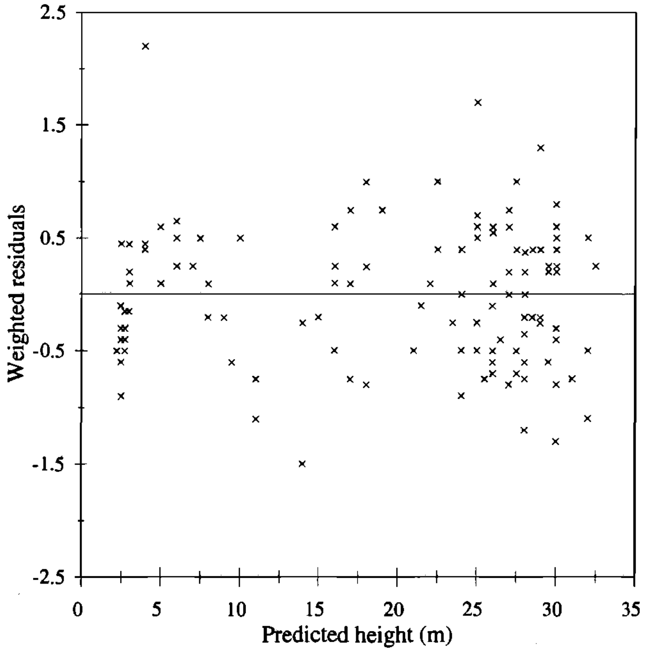

Figure 5. Plot of the weighted residuals versus predicted values for the suggested height–diameter equation.

Age data from the Cuyler stand indicate the potential for growing trees to 40 cm dbh in about 80 yr and 60 cm in a century. These data compare favorably to those reported by Tubbs (1977) for sugar maple after four selection cuttings in a previously unmanaged (virgin) northern hardwood stand (Figure 2). Tubbs' age–diameter relationship indicates that sugar maple trees in the virgin stand grew more slowly than the trees in our stand and maintained a constant rather than increasing growth rate with increasing size. Careful control of spacing within and between age classes should accelerate growth and shorten the time to reach desired threshold sizes. Future evaluations of remeasurement data from permanently marked sample trees will provide a more precise estimate of the rates and patterns of diameter increment following selection system treatments. Yet even a generalized assessment of diameter growth based on the 1973 increment cores suggests that landowners can expect a long-term average rate of at least 6 cm per decade in periodically cut northern hardwoods.

We found considerable variation in height within diameter classes, particularly for larger trees (Figure 4). This likely reflects natural genetic variation, tempered by environmental and other effects on tree development and crown integrity. High stand density prior to the 1973 selection treatment likely resulted in crowding within the upper canopy layers and a deficiency of trees in the smaller diameter classes. This pre-1973 structural imbalance probably affected height and crown development of many trees. Periodic crown breakage during storms and logging also reduced the heights of numerous trees, further increasing the variation among those of similar ages and diameters. Such factors commonly influence the development of trees at high latitudes, causing an overlap of heights between diameter classes. Nevertheless, the regressions show a clear relationship between tree diameter and height in the Cuyler Hill stand. When interpreted in light of the correlation between tree diameter and age, these results suggest a relationship between tree age and height as well.

Future cuttings that leave the most vigorous and highest quality trees of each age class and that regulate spacing and stocking within and among different diameter classes, should reduce the variation in the relationships between tree age and diameter, and tree diameter and height. Yet despite the variability associated with the trees measured for this study, statistically valid and biologically meaningful equations could be fit. These equations provide a suitable means for predicting the age and height of individual trees for most practical applications, and could serve as a basis for descriptive or modeling work in complex uneven-aged stands such as this one. Particular value would accrue from future research that measures all three variables on the same trees, and that follows the development of diameter and height over time in stands under sustained management by single-tree selection system.

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